Optics Communications 102 (1993) 491–494
North-Holland

OPTICS
COMMUNICATIONS

Back-action-evading measurement via nondegenerate optical parametric down-conversion in a ring cavity ☆

Tiancai Zhang, Changde Xie and Kunchi Peng
Institute of Opto-Electronic Research, Shanxi University, Taiyuan, 030006, Shanxi, China

Received 20 April 1993; revised manuscript received 3 June 1993

The optical back-action evading (BAE) measurement via nondegenerate parametric optical down conversion in a ring cavity has been analyzed. Two BAE schemes which are readily performed by experiments are presented and the dependences of BAE gains on the system parameters are discussed.

1. Introduction

Back-action evading measurement is a new technique developed in the 1970s from the necessity of super-low signal detection, especially the gravity-wave detection [1,2]. The back-action-evading measurement would allow one to overcome the perturbation which occurs when an apparatus interacts with a quantum-mechanical system and measures repeatedly the dynamical variables with identical results. Using the squeezed state of light in BAE systems, the signal detection with the precision below the standard quantum limit (SQL) can be accomplished. Because of its potential use in the super-low noise signal detection and optical communication with large capacity, there has been great interest in BAE.

Theoretical researches have demonstrated that the optical BAE technique can be realized by four-wave mixing, optical rectification and optical Kerr medium [3–5]. Yurke [6] presented a variety of optical BAE schemes in 1985. Later, using a single-pass parametric down-conversion process pumped by a pulse laser, Slusher et al. [7] realized experimentally BAE of noise in a quantum measurement of an optical field. Shelby et al. [4] further studied the BAE condition in the parametric amplification process and proposed a BAE measurement scheme in detail. According to this scheme, a mode converter (a Faraday rotator) must be placed in the cavity and it would increase the cavity damping. Meanwhile, it is difficult to ensure that there are the same parametric gains for the signal and readout modes with this scheme. Therefore, experimental demonstration has not been accomplished until now.

The feasibility of BAE via the nondegenerate parametric down-conversion in a ring cavity is studied in this paper. The presented two schemes only include the nonlinear crystal in the cavity without any extra components, so that the cavity damping is reduced to the lowest limit. In this simple system, it is easy to ensure that there are the same gains in signal and readout modes during the parametric interactions.

First of all, we introduce the BAE conditions and then analyze the BAE schemes in ring cavity by means of the mathematical formulas.

2. BAE condition and the proposed BAE schemes

As shown in fig. 1, c^{in}, d^{in}, c^{out} and d^{out} besides labeling optical beam paths also denote the annihilation operators for the modes propagating along the respective paths. According to ref. [6], if the black box can give the transformation

☆ This work was supported by the Chinese National Science Foundation and the Youth Science Foundation of Shanxi Province.

0030-4018/93/$06.00

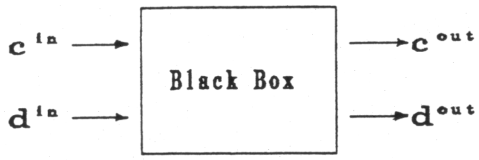

Fig. 1. If the relation of eqs. (1.1) and (1.2) can be given by this black box, the black box is capable of BAE measurement.

$$c^{out}=c^{in}+iG_{BAE}(d^{in}+d^{+in})\ , \tag{1.1}$$

$$d^{out}=d^{in}+iG_{BAE}(c^{in}-c^{+in})\ , \tag{1.2}$$

then the following relations are satisfied:

$$X_c^{out}=X_c^{in}+2G_{BAE}X_d^{in}\ , \tag{2.1}$$

$$Y_c^{out}=Y_c^{in}\ , \tag{2.2}$$

$$X_d^{out}=X_d^{in}\ , \tag{2.3}$$

$$Y_d^{out}=Y_d^{in}-2G_{BAE}Y_c^{in}\ , \tag{2.4}$$

where

$$X_c=(c^{in}+c^{*in})/\sqrt{2}\ , \tag{3.1}$$

$$Y_c=i(c^{in}-c^{*in})/\sqrt{2}\ , \tag{3.2}$$

$$X_d=(d^{in}+d^{*in})/\sqrt{2}\ , \tag{3.3}$$

$$Y_d=i(d^{in}-d^{*in})/\sqrt{2}\ , \tag{3.4}$$

are the quadrature component operators for the c^{in}, d^{in} field modes, respectively.

From eq. (2.4) one sees that Y_c^{in} can be determined by measuring Y_d^{out} (if Y_d^{in} is known), meanwhile, from eq. (2.2) one sees that the component Y_c of the incoming signal is kept clean and this means that BAE detection is realized for Y_c. From eq. (2.1), all the back-action noise is dumped into X_c. The signal-to-noise ratio of the BAE is proportional to G_{BAE} [6]. We call G_{BAE} the BAE gain and the larger the G_{BAE} is, the higher is the detection efficiency.

2.1. Scheme 1

As shown in fig. 2, c^{in} and d^{in} are the incoming signal and readout modes with enter from M_1 and interact with the nonlinear crystal. c^{out} and d^{out} are the output signal and readout modes. M_1 is a partly reflecting mirror used as both input and output mirror. Suppose that M_1 has the same transmission and reflection coefficient t and r for both c and d modes.

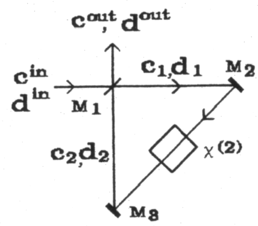

Fig. 2. BAE via the ring cavity.

c_1, d_1 and c_2, d_2 are intracavity annihilation operators before and after the nonlinear medium. M_2 and M_3 are totally reflecting mirrors. One can readily obtain the following equations,

$$c_1=tc^{in}+rc_2\ , \tag{4.1}$$

$$d_1=td^{in}+rd_2\ , \tag{4.2}$$

$$c_2=c_1+iG(c_1+c_1^{+})\ , \tag{5.1}$$

$$d_2=d_1+iG(d_1+d_1^{+})\ , \tag{5.2}$$

$$c^{out}=tc_2-rc^{in}\ , \tag{6.1}$$

$$d^{out}=td_2-rd^{in}\ , \tag{6.2}$$

where G is the single-pass effective gain. From the equations above we can obtain

$$c^{out}=c^{in}+iG(1+r)/(1-r)(d^{in}+d^{in})\ , \tag{7.1}$$

$$d^{out}=d^{in}+iG(1+r)/(1-r)(c^{in}+c^{+in})\ . \tag{7.2}$$

These equations have the same form as eqs. (1.1) and (1.2) and the BAE gain is

$$G_{BAE}=G(1+r)/(1-r)\ . \tag{8}$$

When $r\rightarrow 1$, G_{BAE} approaches ∞, and at a certain reflectivity, G_{BAE} increases linearly as G increases.

2.2. Scheme 2

As shown in fig. 3, M_1 is a partly reflecting mirror and M_2, M_3 are totally reflecting mirrors. P_1 and P_2 are the half-wave plates, which project the fields c^{in} and d^{in} along the polarizations corresponding to the subharmonic fields c', d' of parametric amplifier:

$$c'=\cos\theta\, c^{in}+\sin\theta\, d^{in}\ , \tag{9.1}$$

$$d'=-\sin\theta\, c^{in}+\cos\theta\, d^{in}\ . \tag{9.2}$$

Through mirror M_1, the modes are

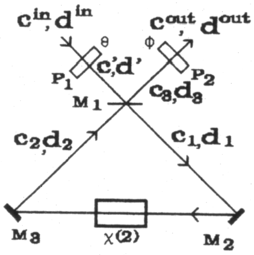

Fig. 3. BAE via the ring cavity and mode converters outside the cavity.

$$c_1 = tc' + rc_2\,, \tag{10.1}$$

$$d_1 = td' + rd_2\,. \tag{10.2}$$

After interacting with the nonlinear crystal, one has

$$c_2 = Gc_1 + gd_1^+\,, \tag{11.1}$$

$$d_2 = Gd_1 + gc_1^+\,, \tag{11.2}$$

$$c_3 = tc_2 - rc'\,, \tag{12.1}$$

$$d_3 = td_2 - rd'\,, \tag{12.2}$$

$$c^{\text{out}} = \cos\varphi\, c_4 + \sin\varphi\, d_4\,, \tag{13.1}$$

$$d^{\text{out}} = -\sin\varphi\, c_4 + \cos\varphi\, d_4\,, \tag{13.2}$$

where $g = \sqrt{1-G^2}$. Substituting eqs. (9) into eqs. (10), one obtains

$$c_1 = t(\cos\theta\, c^{\text{in}} + \sin\theta\, d^{\text{in}}) + rc_2\,, \tag{14.1}$$

$$d_1 = t(-\sin\theta\, c^{\text{in}} + \cos\theta\, d^{\text{in}}) + rd_2\,. \tag{14.2}$$

Respectively substituting eqs. (11) and eqs. (12) into eqs. (14), we have

$$c_1 = t(\cos\theta\, c^{\text{in}} + \sin\theta\, d^{\text{in}}) + r(Gc_1 + gd_1^+)\,, \tag{15.1}$$

$$d_1 = t(-\sin\theta\, c^{\text{in}} + \cos\theta\, d^{\text{in}}) + r(Gd_1 + gc_1^+) \tag{15.2}$$

$$c_3 = t(Gc_1 + gd_1^+) - r(\cos\theta\, c^{\text{in}} + \sin\theta\, d^{\text{in}})\,, \tag{16.1}$$

$$d_3 = t(Gd_1 + gc_1^+) - r(-\sin\theta\, c^{\text{in}} + \cos\theta\, d^{\text{in}})\,. \tag{16.2}$$

Equations (13)–(16) can be solved to express c_1, d_1 in terms of c^{in}, d^{in},

$$c_1 = A[p\cos\theta\, c^{\text{in}} - q\sin\theta\, c^{+\text{in}} + p\sin\theta\, d^{\text{in}} + q\cos\theta\, d^{+\text{in}}]\,, \tag{17.1}$$

$$d_1 = A[-p\sin\theta\, c^{\text{in}} + q\cos\theta\, c^{+\text{in}} + p\cos\theta\, d^{\text{in}} + q\sin\theta\, d^{+\text{in}}]\,, \tag{17.2}$$

where

$$A = t/[(1-rG)^2 - (rg)^2]\,, \tag{18.1}$$

$$p = 1 - rG\,, \tag{18.2}$$

$$q = rg\,, \tag{18.3}$$

and finally we can express c^{out} and d^{out} in terms of c^{in} and d^{in}. The calculation results are

$$c^{\text{out}} = B_1\cos(\theta+\varphi)c^{\text{in}} - B_2\sin(\theta-\varphi)c^{+\text{in}} + B_1\sin(\theta+\varphi)d^{\text{in}} + B_2\cos(\theta-\varphi)d^{+\text{in}}\,, \tag{19.1}$$

$$d^{\text{out}} = -B_1\sin(\theta+\varphi)c^{\text{in}} + B_2\cos(\theta-\varphi)c^{+\text{in}} + B_1\cos(\theta+\varphi)d^{\text{in}} + B_2\sin(\theta-\varphi)d^{+\text{in}}\,, \tag{19.2}$$

where

$$B_1 = tAGp + tAgq - r\,, \tag{20.1}$$

$$B_2 = tAGq + tAgp\,. \tag{20.2}$$

It is readily to testify the following relation,

$$B_1^2 - B_2^2 = 1\,. \tag{21}$$

When

$$\theta = \varphi\,, \tag{22.1}$$

$$\cos 2\theta = 1/B_1\,, \tag{22.2}$$

the overall transformations of the signal and readout input–output quadratures are given by

$$X_c^{\text{out}} = X_c^{\text{in}} \pm 2\sqrt{2(B_1^2-1)}X_d^{\text{in}}\,, \tag{23.1}$$

$$Y_c^{\text{out}} = Y_c^{\text{in}}\,, \tag{23.2}$$

$$X_d^{\text{out}} = X_d^{\text{in}}\,, \tag{23.3}$$

$$Y_d^{\text{out}} = Y_d^{\text{in}} \mp 2\sqrt{2(B_1^2-1)}Y_c^{\text{in}}\,. \tag{23.4}$$

From eqs. (23), the BAE gain is

$$G_{\text{BAE}} = \sqrt{2(B_1^2-1)}\,. \tag{24}$$

Figure 4 shows the relation between the BAE gain G_{BAE} and the single-pass gain G. The dashed line corresponds to the first scheme. The G_{BAE} increases linearly with G. The solid curve corresponds to the second scheme. Although in most cases there is low G_{BAE}, for some certain G, much larger G_{BAE} than that in first scheme will be obtained. The angles rotated by P_1 and P_2 are the same according to our schemes. For certain system parameters, a certain rotating angle can be chosen to satisfy the BAE condition and

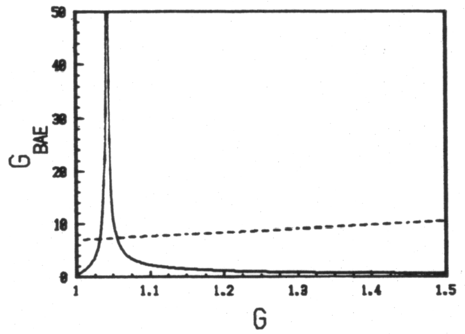

Fig. 4. The relation between the BAE gain and the single pass gain ($r=0.75$).

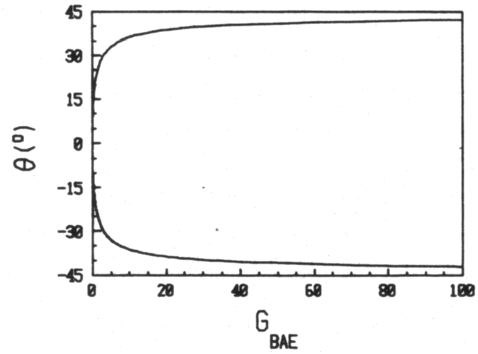

Fig. 5. The relation between the BAE gain and the rotating angle ($r=0.9$).

get large G_{BAE}. From fig. 5, we can see that, for any BAE gain given by system parameters through eq. (24) and (20.1), a certain angle between $0° \sim +45°$ (or $0° \sim -45°$) can be chosen to satisfy the BAE condition.

It should be noticed that the transformations of eqs. (5) and eqs. (11) are given on the proper pump phases [6]. We have provided a simple analysis for these phases. If the pump phase noise is considered, these transformations are approximately satisfied, and the BAE precision is reduced. The quantitative analysis has gone beyond this paper and we would like to discuss this subject in our forthcoming publication.

3. Conclusion

The possibility of performing optical BAE through a parametric process in a ring cavity has been studied. The schemes presented are simple and reliable. The first scheme is very simple but the variation of G_{BAE} with G is small. At a given single-pass gain of crystal, the second scheme can yield large G_{BAE} when choosing appropriate rotating angles. Both of these schemes overcome the shortcoming brought from the intracavity mode converter and are the feasible proposals to realize the cw BAE measurement.

References

[1] V.B. Braginsky and Yu.I. Vorontosov, Usp. Fiz. Nauk. 114 (1974) 41.

[2] K.S. Thorne, R.W.P. Drever, C.W. Caves, M. Zimmermann and V.D. Sandberg, Phys. Rev. Lett. 40 (1978) 667.

[3] G.J. Milburn and D.F. Walls, Phys. Rev. A 28 (1983) 2065.

[4] R.M. Shelby and M.D. Levenson, Optics Comm. 64 (1987) 553.

[5] H.A. Bachor, M.D. Levenson, D.F. Walls, S.H. Perlmutter and R.M. Shelby, Phys. Rev. A 38 (1988) 180.

[6] B. Yurke, J. Opt. Soc. Am. B 2 (1985) 732.

[7] A. La Porta, R.E. Slusher and B. Yurke, Phys. Rev. Lett. 62 (1989) 28.